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INVESTIGATION OF HEAT TRANSFER IN THE CORE OF THE OAK RIDGE HOMOGENEOUS REACTOR

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INVESTIGATION OF HEAT TRANSFER IN THE CORE OF THE OAK RIDGE HOMOGENEOUS REACTOR

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SUMMARY

In the Homogeneous Reactor Test (HRT) at Oak Ridge National Laboratory (ORNL), the reactor core vessel is constructed of Zircaloy -2. This material is subject to corrosion by the core fluid during reactor operation. Previous experimental studies indicate the corrosion rate is doubled for every 25°C increase in temperature. Consequently, the temperature of the core wall is important. Also, to prolong the life of the core vessel, cooling of the wall is considered feasible. Since no experimental data were available, the purpose of this investigation was to determine the core wall temperature as a function of the heat transfer rate at the outer wall by analytical methods.

The work reported here was not intended to be a complete analytical solution to the problem. Instead, the study was performed to obtain values which would be representative of those in the actual case. The method of attack was to initially define a simple model of the actual system so that previous related analytical solutions could be utilized. Simplifying assumptions were made to idealize the system. Approximate analytical expressions were derived which permitted calculation of the desired results.

The flow regime in the reactor core was determined to be turbulent. The model of the system was a series of short pipes. An analysis on the basis of forced convection heat transfer in turbulent flow revealed that free convection effects could not be considered negligible.

For the same model of the system, results were obtained for combined free and forced convection heat transfer. The analysis in this case was based on laminar flow. A comparison of the results of the two cases was made for high heat transfer rates. When the heat transfer rate is high, free convection effects in the system become negligible. The comparison showed the results to be in close agreement and it was concluded that the analysis on the basis of laminar flow yielded reasonable results. The results were presented in graphical form.

The results obtained in this study are only applicable in the upper hemisphere of the core. Under present design conditions, the results indicate that wall temperature less than 15°C above the mean core fluid temperature will prevail. Also, reduction of the wall temperature was found possible provided high, but not unreasonable, rates of heat transfer can be obtained by modification of design. For example, at the equator of the core, the inside wall temperature can be reduced approximately 25°C below the mean fluid temperature for a heat transfer rate of 15,000 BTU/HR-FT^2 at the outside wall.

In the lower hemisphere of the core, a possibility of hot spots on the wall was noted. A recommendation was given that this problem be investigated experimentally. The assumption of the Prandtl modulus equal to one was also recommended for particular cases involving heat transfer investigations of aqueous homogeneous reactors. The suggestion of a method for reducing experimental difficulties associated with heat generating fluids was given. No experimental data were available to confirm the results presented in this report. Therefore, it was advised that caution be exercised in the application of the results.

CHAPTER I

INTRODUCTION

Statement of the problem.--The core vessel of the Homogeneous Reactor Test (HRT) at Oak Ridge National Laboratory is constructed of Zircaloy-2 and is subject to corrosion by the core fluid. The rate of corrosion is approximately doubled for every 25°C increase in temperature (1)* within the range of the reactor operation. Consequently, the temperature distribution as a function of position on the core vessel wall is important. In an attempt to lower the core wall temperature and thus prolong the life of the reactor, it is feasible that modifications in design be made. On this basis, the heat transfer rate necessary to maintain wall temperatures within prescribed limits becomes desirable.

Object.--The purpose of this study is to examine the mechanisms of fluid flow and heat transfer in the reactor core vessel, especially in the vicinity of the vessel wall. The desired result is to establish the magnitude of the heat transfer rate at the core wall as a function of temperature of the wall.

Scope.--The work reported here is not intended to be a complete solution of the problem. Instead, a study of available information related

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*Numbers in parenthesis refer to references listed in the bibliography.

to the problem is to be presented which will enable numerical results to be obtained. The actual physical system will be idealized to a simple model, which will permit previous analytical solutions to be used to make computations. Finally, with all simplifying assumptions clearly stated, the results of the computations will be presented. The results presented should be indicative of actual values that will be obtained in the reactor test.

CHAPTER II

SURVEY OF RELATED INVESTIGATIONS

Fluid flow within a spherical container.--The velocity and temperature distributions in a spherical vessel containing a fluid with uniform volume heat sources have been examined analytically by Fax (2). In this work, the case of potential flow was considered. Frictional effects and diffusion of heat were neglected. No direct application to the problem in the HRT was found for the results of the reported work.

Boarts and Peebles (3) performed a combined analytical and experimental study on liquid flow in a spherical system. The problem considered a liquid entering a spherical container at the equator and leaving at both the north and south poles of the vessel. Again, the results were not found to be applicable to the present problem.

Hydrodynamic tests of limited extent have been carried out by Harley (4) on a full scale mock-up of the reactor core. Water at approximately room conditions was circulated through a test loop containing the vessel. The design flow rate was maintained during the test and the fluid was found to sweep the entire vessel with no observable stagnant regions. Velocity data by the salt-solution method were obtained and observations of the flow pattern were made by injection of a dye in the flow stream. Since the tests were made at pressures and temperatures below those for operating conditions and with no heat generation in the fluid, the results are not strictly applicable to this study. However, these tests do provide valuable information concerning the flow regime to be expected in the core of the reactor during operation.

Reactor core temperatures.--An investigation of the temperature distribution in the homogeneous reactor core wall has been carried out by Haubenreich (5). The problem was simplified by using a flat plate model of the system. The heat generation rate in the wall was determined and free convection equations for vertical and horizontal plates were employed for calculations. With these results, he was able to predict the temperature distribution in the wall for three positions on the core vessel. Heat generation in the core fluid was not considered. Also, temperatures at the inside core wall were not examined for variable heat transfer rates at the outside surface of the vessel.

Heat transfer in ducts with heat generation in the fluids.--Poppendiek and Palmer (6, 7, 8) have investigated forced convection heat transfer with uniform heat sources in the fluids. An analytical study of combined free and forced convection heat transfer in laminar flow has been completed by Hallman (9). The results of these investigations were extensively employed in the present study and will be discussed in detail in a subsequent chapter of this report.

Hamilton and co-workers (10, 11) have examined the case of free convection heat transfer with volume heat sources. These works include both theoretical and experimental studies and the results should be applicable to the problem in the HRT. Ostrach (12) performed an analytical examination of laminar free convection heat transfer with and without heat sources in channels of constant wall temperature. Also reported by Ostrach (13) was the investigation of combined free and forced convection heat transfer in laminar flow with and without heat sources in channels with linearly varying wall temperatures. The results of these works are of considerable interest in the present problem, but due to the limited scope of this study, application of the results was not possible.

Several other investigations that would be of interest in a more thorough analysis of the problem are listed in the bibliography.

CHAPTER III

PHYSICAL SYSTEM

Description.--The reactor is designated as an aqueous homogeneous reactor. Uranyl-sulfate is dissolved in heavy water to give a homogeneous combination of a heat source and moderator. In the reactor core section, the core containing the uranyl-sulfate solution is blanketed by a heavy water region which acts as a reflector. The fuel solution enters at the bottom of the core vessel, flows through two diffuser sections and the main spherical shaped section and leaves at the top of the container. Details of the reactor are reported by Kasten (14) and an illustration of the core vessel is given in Figure 1. Pertinent design data for the reactor are given in the appendix.

Fluid mechanics.--The fuel solution of the homogeneous reactor may be considered as an incompressible fluid. The motion of the fluid in the reactor core is three-dimensional. The solution of mathematical expressions describing the motion of the fluid has not been accomplished and will not be attempted in this study. Consequently, analytical descriptions of the pressure and velocity distribution in the particular case of the HRT core vessel are not available.

Experimental information on the fluid velocity in the core is notably lacking. The only data available are from reference (4) and the tests from which the data were obtained were made at room conditions.

However, adequate information is available to establish the flow regime that will occur in the core during reactor operation.

The flow regime is generally determined by calculating the Reynolds modulus for the system and comparing it with a critical value established by previous experiments involving the type of system under consideration. If the Reynolds modulus for the system is less than the critical value, usually taken as 2100 for circular pipes, laminar flow is presumed to occur. For values exceeding the critical value, turbulent flow is assumed. Transition flow is believed to occur at values near the critical Reynolds modulus. This regime of flow is not well defined and depends on several factors within a particular flow system, such as initial disturbance, entry length of the flow duct and surface roughness of the duct.

The HRT core vessel is not a conventional flow duct and hence no critical Reynolds modulus has been determined. The cross-sectional area of flow is always circular so some similarity to a pipe system exists. When a Reynolds modulus is calculated for the reactor core on the arbitrary basis of using the flow cross-sectional diameter as the significant length parameter, a comparison may be made with a conventional pipe system. Calculations for the reactor core at operating conditions reveal that the critical Reynolds modulus is exceeded for all sections of the vessel. This indicates turbulent flow will probably occur, although other factors need to be considered.

The entry length for the flow system in the reactor cannot be clearly defined. However, a comparison of the system with a pipe suggests that an adequate entrance section for fully developed flow is not available. Nevertheless, the diffuser screens and diverging section of the lower part of the core act as turbulence promoters. These factors and the Reynolds modulus are sufficient evidence to predict that the flow regime in the reactor core during operation will be fully turbulent.

Experimental results tend to substantiate the prediction of fully turbulent flow in the core. The tests reported in reference (4) indicate that turbulence was present to some extent even at room conditions. The Reynolds modulus at operating conditions is approximately five times that at room conditions on the basis that the kinematic viscosity of the core fluid decreases by a factor of five from ambient conditions to operating conditions while other parameters remain constant. This increase in the Reynolds modulus, plus other factors associated with heat sources in the core fluid, will promote turbulence to an extent beyond that observed in tests. The conclusion is that fully turbulent fluid motion will prevail in the reactor core during normal operation.

<u>Heat transfer (internal)</u>.--Several factors must be taken into account in a description of the mechanism of heat transfer in the homogeneous reactor. For instance, the rate of heat generation in the core fluid as well as the spatial distribution of the heat sources is of importance. Since the fluid is pumped through the core, there will be forced convection heat transfer. Also, due to heat sources in the fluid and core wall, density variations will occur and free convection effects need to be considered. All of the above factors are interrelated to the

temperature distribution in the core. The following is an attempt to present a qualitative discussion of the factors relating to heat transfer in the core of the reactor.

The heat generation rate is known to be a maximum at the center of the core and to decrease in the radial direction to about sixty-five per cent of the maximum at the core wall (15). The heat generation rate in the core wall is available from reference (5). From this information there is a possibility of three forms of the velocity and temperature distribution. First, if heat is removed at the outside wall of the core at the same rate it is generated in the wall, then the system may be considered to have an adiabatic wall with respect to the fluid in the core. Provided the rate of heat transferred out of the wall (to the reflector fluid or coolant) exceeds the rate of heat generation in the wall, the fluid in the core may be assumed to have a cooled wall. For the case of no heat transfer at the outer wall, the heat generated in the wall will be transferred to the core fluid under steady state conditions and this presents a heated wall with respect to the core fluid. Figure 2 shows qualitatively the shape of the velocity and temperature profiles for the three cases just mentioned.

Due to the large dimensions of the core vessel, the effects of heat transfer at the wall will probably be appreciable only in the region of fluid near the vessel wall. In the case where heat is added to the fluid through the vessel wall, there will be a large temperature drop from the wall to the bulk of the fluid. The major portion of this temperature difference will occur in the fluid very near the wall. The temperature near the wall will be higher than the mean temperature of the fluid so that the viscosity of the wall fluid will be reduced. Free convection effects will then cause an increase in velocity near the wall with the consequence that high coefficients of heat transfer may be expected. The same will be true for the adiabatic wall case except to a lesser extent.

The case of wall cooling will result in an increase in the viscosity of the fluid near the wall. Therefore, the increased frictional forces in the fluid will reduce the velocity of the fluid in the vicinity of the wall. Free convection effects may be considered negligible in this case. The low velocity region of fluid will have less of the heat generated within the fluid convected away from the wall. In steady state, a high rate of heat transfer will be required to maintain the wall at a temperature below the bulk fluid temperature. High heat transfer rates for wall cooling may be difficult to attain since the heat transfer coefficient in this case is not expected to be very large.

The preceding discussion will not apply to the entrance section of the core where the diffuser screens are located. Also, in the diverging section directly above the screens, conditions will not be the same as for the cases discussed above. In the flow tests reported by Harley (4) separation of the flow at the wall was observed to occur in the lower hemisphere of the core. At the point of separation there is a tendency for stagnation which will result in a high temperature region. Whether or not this will have a significant effect on the wall temperature has not been determined. The possibility of transient hot spots at the core wall should be noted, however, since an equilibrium temperature structure in this sector of the core is questionable.

High heat transfer coefficients could prevail in the lower part of the core on the basis that it acts as a thermal entrance region. No prediction can be made since information on thermal entrance regions involving heat generating fluids could not be found. An analysis of this problem is beyond the scope of this work.

Heat transfer (external).--The homogeneous reactor is designed to transfer the heat generated in the core to a secondary fluid at a location removed from the core region. Nevertheless, some heat transfer from the core fluid through the wall to the reflector fluid will occur provided a negative temperature gradient through the wall exists. Due to the large volume and low flow rate of the reflector fluid, the mode of heat transfer will be predominantly free convection. The design temperature of the reflector fluid is only slightly lower than the mean temperature of the core fluid (14). Therefore low heat transfer coefficients may be expected at the outside core wall.

CHAPTER IV

ANALYSIS

Preliminary considerations.--The attempt to obtain a solution of the problem involved many simplifying assumptions. The assumptions made in the approach to the problem were based on both the physical system and on the conditions required for application of previous solutions of related investigations. Justification is not satisfactory in all cases. However, in view of the complexity of the overall problem and the lack of experimental data, simplifications could not be avoided.

Prandtl modulus of the homogeneous reactor fuel solution.--The Prandtl modulus is a group of variables that are always important in both free and forced convection heat transfer. It may be expressed as

$$Pr = \frac{\mu c_p}{k} \tag{1}$$

where Pr = Prandtl modulus

μ = viscosity

c_p = specific heat at constant pressure

k = thermal conductivity

The fuel solution for the homogeneous reactor is a ten gram per kilogram solution of uranyl-sulfate in heavy water. For the approximate operating conditions of the reactor of 2000 PSIA and 536°F, the property values of the fuel solution are available from reference (14). Substitution of the appropriate values into Equation (1) yields the result of Pr = 0.92. On this basis, the assumption is made that the Prandtl modulus equals one.

It is worthy of note that the assumption of a Prandtl modulus equal to one is probably justifiable for most fuel solutions for aqueous homogeneous reactors. Property values for fuel solutions are not known within the entire range of possible reactor operation. However, from the data available for the HRT, it is suspected that the Prandtl modulus of homogeneous reactor fuel solutions does not vary significantly from that of light water at the same conditions.

Values of the Prandtl modulus of light water in the range of possible reactor operation are available from reference (16). The values are plotted in Figure 3 for the cases of saturated liquid and of liquid water under a pressure 6000 PSIA. Values for intermediate pressures lie between the two curves shown in the figure. The range of temperature is from 300°F to 600°F. Also indicated in Figure 3 is the Prandtl modulus for the HRT fuel solution. The figure shows that water, under the conditions considered, has a Prandtl modulus of approximately one. Therefore, in cases where the Prandtl modulus of the fuel solution is not known, the assumption of Prandtl modulus of unity appears to be a good approximation, provided the HRT fuel solution is a typical example.

Simplified model of the system.--The initial approach to the problem was to define a simple model of the actual system. Examination of the physical system revealed that two reasonable models of the system, the pipe model and the flat plate model, could have been defined. The pipe model appeared applicable since the cross-sectional area is always circular in the direction of net flow. The radius of curvature of the core vessel is large in comparison with ordinary pipes, so the core wall could be considered a flat plate. When available analytical solutions involving the two models were taken into account in the choice of the model, the pipe system seemed the best selection.

The pipe model of the homogeneous reactor core is defined by consideration of one-dimensional flow through a very short horizontal section of the core vessel. In this section the fluid motion and the mode of heat transfer are assumed to be the same as for a circular pipe of the same diameter with similar boundary conditions. At the extreme sections in the upper and lower hemisphere of the core, one-dimensional flow cannot reasonably be assumed. Consequently, the model of the system is only defined for a certain portion of the core. The region for which the model is defined is arbitrarily selected as extending from the equator of the core to the sections where the cross-sectional radius is one foot.

<u>Forced convection heat transfer</u>.--The case of forced convection heat transfer in turbulent flow in pipes with volume heat sources in the fluid has been thoroughly analyzed by Poppendiek and Palmer (6). In order that their results may be applied to the present problem, the postulates made to idealize the system for analysis must also be applied to the model of the present system. The postulates are

(a) Thermal and hydrodynamic patterns have been established

(b) Uniform volume heat sources exist within the fluid

(c) Physical properties are not functions of temperature

(d) Heat is transferred uniformly to or from the fluid at the pipe wall

(e) The generalized turbulent velocity profile defines the hydrodynamic structure

(f) There exists an analogy between heat and momentum transfer.

For turbulent fluid motion, the differential equation relating the variables of the idealized system is

$$\frac{u}{u_m} \frac{q''' + \frac{2}{r_o} q''_o}{\rho c_p} = \frac{1}{r} \frac{d}{dr} \left[(\alpha + \epsilon) r \frac{dt}{dr} \right] + \frac{q'''}{\rho c_p} \qquad (2)$$

where u = local velocity in flow direction

u_m = mean velocity in flow direction

r_o = inside radius of pipe

q''' = volume heat source in core fluid

q''_o = heat transfer rate per unit area at inside wall surface

ρ = fluid density

c_p = specific heat at constant pressure

r = radial distance from centerline of pipe

α = thermal diffusivity

ϵ = eddy diffusivity

t = local temperature at radius r

Equation 2 may be separated into two simpler equations, solutions of which can be superposed to yield the solution of the original equation. These equations are

$$\frac{u}{u_m}\frac{q'''}{\rho c_p} = \frac{1}{r}\frac{d}{dr}\left[(\alpha + \epsilon)\, r\,\frac{dt}{dr}\right] + \frac{q'''}{\rho c_p} \tag{3}$$

and

$$\frac{u}{u_m}\frac{2q''_o}{r_o \rho c_p} = \frac{1}{r}\frac{d}{dr}\left[(\alpha + \epsilon)\, r\,\frac{dt}{dr}\right] \tag{4}$$

Equation 3 describes a system with uniform volume heat sources but no wall heat flux. The system described by Equation 4 has uniform wall heat flux with no heat sources in the fluid. The solution of Equation 3 is due to Poppendiek and Palmer (6). Martinelli (17) is responsible for the solution of Equation 4.

Approximation of the results of the solutions of these equations may be made on the basis of Prandtl modulus equal to one. The results of the solution of Equations 3 and 4 for the mixed mean fluid temperature minus the inside wall temperature are presented in graphical form in references (6) and (17). The curves on these graphs for a Prandtl modulus of one are approximately represented by the equations

$$(t_m - t_o)_{VHS} = -60\,\frac{q''' r_o^2}{k}\,Re^{-1.13}\,; \quad \left[Pr = 1\right] \tag{5}$$

and

$$(t_m - t_o)_{NO\ VHS} = 60 \frac{q_o'' r}{k} Re^{-0.755} ; \left[Pr = 1 \right] \tag{6}$$

where t_m = mixed mean fluid temperature

t_o = inside wall temperature

k = thermal conductivity of the fluid

Re = Reynolds modulus

VHS = refers to volume heat sources in the fluid

NO VHS = refers to no volume heat sources in the fluid.

Equation 5 represents an approximation of the results of the solution of Equation 3 and Equation 6 is an approximation of the results of the solution of Equation 4. Figure 4 presents the actual graphical results of the solution of Equation 3 and a curve of Equation 5. Similarly, Equation 6 is shown in Figure 5 with the actual curve which is approximated.

Superposing Equations 5 and 6 yields an approximate solution of Equation 2. The result is

$$(t_m - t_o) = 60 \frac{r_o}{k} Re^{-0.755} \left[q_o'' - q''' r_o Re^{-0.375} \right] ; \quad \left[Pr = 1 \right] \tag{7}$$

In the above equation, the temperature difference is given in terms of q_o'', the heat flux at the inner wall. To account for the effects of heat generation in the wall on the temperature difference, a slight modification of the equation is necessary. If there exists an additional heat flux at the wall due to heat sources in the wall, Q'', then the outside wall heat flux, q''_w, becomes

$$q''_w = q''_o + Q'' \tag{8}$$

The value of q''_o from Equation 8 may be substituted into Equation 7 to give

$$(t_m - t_o) = 60 \frac{r_o}{k} Re^{-0.755} \left[q''_w - Q'' - \left(q''' r_o\right) \left(Re^{-0.375}\right)\right] ; \quad \left[Pr = 1\right] \tag{9}$$

Figure 6 shows the results of calculations based on Equation 9 for the homogeneous reactor. The constants employed in the computations are given in the appendix.

Combined free and forced convection heat transfer.--The results of forced convection heat transfer calculations indicate the necessity for consideration of free convection effects. The high temperature differences shown in Figure 6 would create density variations in the fluid with the consequence of higher velocities near the wall. An increase in the velocity of the fluid near the wall would result in a greater amount of heat convected downstream and, therefore, lower wall temperatures than are predicted when forced convection alone is considered.

Although the prediction of turbulent flow has been made, the analysis here is based on a laminar flow regime. The assumption is made that values obtained from consideration of both free and forced convection heat transfer will be more accurate than for the case where natural convection effects are neglected. The analytical solution of the problem of combined free and forced convection heat transfer with volume heat sources in the fluid has only been carried out for the case of laminar flow. Hallman's (9) results for the problem will be utilized to make computations. Discussion of the effect of applying the results of a laminar flow analysis to a turbulent flow problem will be given subsequently.

The model of the reactor core described earlier is again employed with the following additional assumptions from reference (9):

(a) The temperature and velocity profiles are fully developed

(b) No turbulence is present

(c) The flow is axially symmetric with the axis of the pipe lying parallel with the body force

(d) Fluid properties are constant except for density which is allowed for by considering the thermal coefficient of volume expansivity

(e) Heat is transferred uniformly to or from the fluid at the pipe wall

(f) The rate of heat generation per unit volume is uniform everywhere in the fluid.

The momentum and energy equations in differential form for the system are

$$\frac{\partial p}{\partial x} + \rho = \frac{\mu}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r}\right) \tag{10}$$

and

$$u \frac{\partial t}{\partial x} = \alpha \left(\frac{1}{r} \frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial r^2} + \frac{\partial^2 t}{\partial x^2}\right) + \frac{q'''}{\rho c_p} \tag{11}$$

where

where	p	=	pressure
	x	=	distance measured along axis of pipe upward
	ρ	=	fluid density
	μ	=	viscosity
	r	=	radial distance from centerline of pipe
	u	=	fluid velocity parallel with pipe axis at radius r
	t	=	temperature
	α	=	thermal diffusivity
	q'''	=	volume heat source term
	c_p	=	specific heat at constant pressure

From the solution of the above equations, Hallman (9) gives the mixed-mean-to-wall temperature difference as

$$(t_m - t_o) = \frac{q''' r_o^2}{4k} \left[\frac{64C}{\lambda^4} \left(mn + \frac{2m}{\lambda} - 1\right) \right. \tag{12}$$

$$\left. + \left(\frac{1}{F\lambda^4} - \frac{64FC}{\lambda^8}\right) \left(2\lambda^2 n^2 - 2\lambda^2 m^2 + 8\lambda n\right) \right]$$

where t_m = mixed mean fluid temperature

t_o = inside wall temperature

r_o = inside radius of pipe

k = thermal conductivity of fluid

u_m = mean fluid velocity

β = thermal coefficient of volume expansivity

q_o'' = heat flux at inside wall

ν = kinematic viscosity of the fluid

$$\lambda = (Gr)^{\frac{1}{4}} \quad ; \quad \left[Pr = 1\right] \tag{12a}$$

$$Gr = \frac{\beta r_o^4 q'''F}{\rho \nu^2 u_m c_p} \tag{12b}$$

$$F = 1 - \frac{2q_o''}{q'''r_o} \tag{12c}$$

$$m = \frac{\mathrm{ber}_o\lambda \ \mathrm{bei}_o'\lambda - \mathrm{bei}_o\lambda \ \mathrm{ber}_o'\lambda}{\mathrm{ber}_o^2\lambda + \mathrm{bei}_o^2\lambda} \tag{12d}$$

$$n = \frac{\mathrm{ber}_o\lambda \ \mathrm{ber}_o'\lambda + \mathrm{bei}_o\lambda \ \mathrm{bei}_o'\lambda}{\mathrm{ber}_o^2\lambda + \mathrm{bei}_o^2\lambda} \tag{12e}$$

$$C = \frac{\lambda^2}{16}\left[\frac{\lambda}{n}\left(1 - \frac{1}{F}\right) + \frac{2m}{nF}\right] \tag{12f}$$

Computations for the HRT employing Equation 12 yielded results which are presented in graphical form in Figure 7.

Comparison of forced convection heat transfer with and without volume heat sources in the fluids.--In the development of reactor systems, extreme difficulties are encountered in experiments to obtain data for actual reactor operating conditions. A possible method for the reduction of experimental difficulties may be noted from previous results of this study. The investigation of forced convection heat transfer yielded approximate equations for the case of no heat sources in the fluids and for the case of heat sources in the fluids. A ratio of the temperature difference for both cases is found by dividing Equation 7 by Equation 6. The result is

$$\frac{(t_m - t_o)^*}{(t_m - t_o)_{\text{NO VHS}}} = 1 - \frac{r_o q'''}{q''_o} \text{Re}^{-\frac{3}{8}} \; ; \left[\text{Pr} = 1\right] \tag{13}$$

where the superscript (*) refers to the combined case of heat transfer at the wall and heat sources in the fluid.

By definition,

$$\text{Nu} = \frac{2r_o q''_o}{k(t_m - t_o)_{\text{NO VHS}}} \tag{14}$$

where Nu is the Nusselt modulus.

Also

$$Nu^* = \frac{2r_o q_o''}{k(t_m - t_o)^*} \tag{15}$$

From these relations it is obvious that

$$\frac{Nu}{Nu^*} = \frac{(t_m - t_o)^*}{(t_m - t_o)_{NO\ VHS}} = 1 - \frac{r_o q'''}{q_o''} Re^{-\frac{3}{8}} \quad ; \tag{16}$$

$$\left[Pr = 1\right]$$

Heat transfer experiments involving no volume heat sources would permit the determination of Nu. For the same system and the same heat flux at the wall, the value of Nu^* could be obtained from Equation 16. This analysis is, of course, for a special case and alone of little value in relation to the problem encountered in a complete reactor system. However, similar analyses for other cases should provide sufficient information to considerably reduce the difficulties of obtaining experimental data for homogeneous reactors.

CHAPTER V

DISCUSSION OF RESULTS

The assumption of a Prandtl modulus of unity was a valuable aid in the simplification of this study. Some of the approximate equations derived earlier in the report illustrate the value of this assumption for analytical investigations. In the present study, data were available to justify the assumption made. The prediction that the assumption may be made for certain homogeneous reactor fuel solution is not so well founded. However, in investigations involving air at atmospheric conditions, the same assumption is commonly made, although the Prandtl modulus is known to be 0.7. For fuel solutions used in aqueous homogeneous reactors, the error involved in making the assumption, at reactor operating conditions, should compare favorably with that in which air is the fluid.

The considerable practical importance of a Prandtl modulus equal to one may be further emphasized by the following specific example. Parallel flow past a flat plate at zero incidence is the example and is given by Schlichting (18). With proper assumptions, the following differential equations result:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \qquad (14a)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \tag{14b}$$

$$u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y} = \alpha \frac{\partial^2 t}{\partial y^2} \tag{14c}$$

The velocity field is independent of the temperature field so that the two flow equations (Equations 14a, b) can be solved first and the result employed to evaluate the temperature field. However, if the properties of the fluid satisfy the relationship

$$\nu = \alpha \ , \quad \text{i.e. Pr} = 1$$

then the velocity and temperature distributions are identical.

The preceding example is only one of many cases in which Prandtl modulus equal to one results in simplification of the problem. The assumption of the Prandtl modulus equal to one for aqueous homogeneous reactor fuel solutions should prove to be of practical importance.

The simplified model of the physical system, together with the assumption of a Prandtl modulus equal to unity, was employed to obtain results for forced convection heat transfer. The simple model is expected to be valid only for the upper hemisphere of the core vessel. The effects of the diffuser screens and the divergent section of the lower hemisphere are contrary to the postulates made in the analysis by Poppendiek and Palmer (6) upon which the results for this case were based. The model in the upper hemisphere should comply with these postulates except for uniform heat generation in the fluid. Since the major considerations in this study were effects at the core wall, the heat source term was considered constant at the value predicted at the core wall. The results of the forced convection heat transfer analysis are obviously not indicative of the actual case in the reactor because of the high temperature differences predicted. The high temperature differences point out the need for consideration of free convection effects.

The values for wall temperatures in the combined free and forced convection case may be compared with the results for forced convection alone. When a high heat transfer rate at the wall occurs, the effects of free convection may be assumed negligible. Under this condition, a comparison of the two cases should yield values of the same order of magnitude. The results presented in Figures 6 and 7 reveal this to be true. At the extreme section considered for the upper hemisphere, the temperature differences between the wall and the mixed-mean value of the fluid for both cases are around 35°C when the heat transfer rate is 20,000 BTU/HR.FT^2. Since the forced convection analysis considered the proper flow regime, it is believed to be valid when free convection effects are negligible, especially at the upper section of the core. With this consideration, the above comparison leads to the conclusion that the values obtained for the combined free and forced convection case are at least reasonable.

The desired result of this investigation was the inside core wall temperature as a function of the heat transfer rate at the outer wall. Figures 6 and 7 present this result provided the mean fluid temperature is known. From reference (14), the design values for the HRT fluid temperature at entrance and exit to the reactor core are given as 256°C and 300°C, respectively. In this study, the value of the mean fluid temperature at the core equator was taken as 280°C and at the upper section of the vessel as 295°C. These values were combined with previous temperature differences to give the results shown in Figure 8. In this figure, only the combined free and forced convection results are shown.

The results presented in Figure 8 indicate that core wall temperatures will not be excessive in the upper hemisphere. Also, the results show that if high heat transfer rates can be obtained at the outside wall, then inside wall temperatures can be considerably reduced.

Experimental data are not available to confirm the results of this study. A comparison was made, however, with values computed in a similar study of the same problem and reported by Haubenreich (5). In that work a different wall thickness was considered and corrections for this factor were made for comparison. The resulting values were in good agreement (see Figure 8) with those of the present work. Predicted wall temperatures from reference (5) were slightly lower, but this was expected since heat sources in the fluid were neglected in that investigation.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Conclusions. --The flow regime in the core of the homogeneous reactor is expected to be turbulent at operating conditions of the reactor. In the lower hemisphere of the core, a stable flow pattern is questionable with the possibility that transient hot spots at the core wall may exist in this sector. The mode of the heat transfer at the inside core wall is predicted to be combined free and forced convection, with free convection effects predominant at low heat transfer rates. On the basis of a simplified analysis, temperatures at the inside core wall of the upper hemisphere were determined to be approximately the same as the mean fluid temperature. The wall may be 10°C to 15°C above the temperature of the fuel solution, the exact value being dependent on the heat transfer rate at the outside wall. Reduction of the inside wall temperature appears possible provided high heat transfer rates can be obtained by modification of the present design. For the present design, coefficients of convection heat transfer are expected to be low at the outer wall compared to values inside the core vessel.

Recommendations. --Since no experimental data are available to check the results found in this study, caution should be exercised if the results are to be applied. It is strongly recommended that tests be performed, not specifically to confirm the results of this report, but to examine conditions in the lower hemisphere of the core. In this sector the rate of corrosion may be decreased by a reduction of the wall temperature. Hence, if it is desired to increase the life of the reactor core vessel, a method for obtaining high heat transfer rates at the core wall should be devised. Several methods are possible. For example, the flow rate of the reflector fluid could be increased, the reflector fluid temperature could be lowered, or the fluid could be made to flow in the blanket region in the opposite direction of the fuel solution so that a counterflow heat exchange occurred. Structural changes could also be made such that a higher heat transfer rate could be achieved. A combination of these would probably represent the best method.

APPENDIX

NOMENCLATURE

α	=	thermal diffusivity, FT^2/HR
$ber_o\lambda$, $bei_o\lambda$	=	$J_o(i^{\frac{3}{2}}\lambda) = I_o(i^{\frac{1}{2}}\lambda) = ber_o\lambda + i\,bei_o\lambda$
$ber'_o\lambda$	=	$\frac{d}{d\lambda}(ber_o\lambda)$
$bei'_o\lambda$	=	$\frac{d}{d\lambda}(bei_o\lambda)$
$ber_o^2\lambda$, $bei_o^2\lambda$	=	$(ber_o\lambda)^2$, $(bei_o\lambda)^2$
β	=	thermal coefficient of volume expansivity, 1/°F
C	=	defined by Equation (12f)
c_p	=	specific heat at constant pressure, BTU/LB.°F
ϵ	=	eddy diffusivity, FT^2/HR
F	=	defined by Equation (12c)
g	=	acceleration due to gravity, FT/HR^2
I_o	=	modified Bessel function of first kind of order O
i	=	$\sqrt{-1}$
J_o	=	Bessel function of first kind of order O
k	=	thermal conductivity, BTU/HR,FT.°F
λ	=	defined by Equation (12a)
m	=	defined by Equation (12d)
μ	=	dynamic viscosity, $LB/HR.FT^2$
n	=	defined by Equation (12c)

ν	=	kinematic viscosity, FT^2/HR
q''	=	heat transfer rate per unit area, $BTU/HR.FT^2$
Q''	=	heat transfer rate per unit area due to heat generation in core wall, $BTU/HR.FT^2$
q'''	=	heat generation rate, $BTU/HR.FT^3$
r	=	radial distance measured from vertical centerline in a horizontal plane, FT
ρ	=	density, LB/FT^3
t	=	temperature, °F
u	=	velocity in x-direction, FT/HR
v	=	velocity in y-direction, FT/HR
x	=	distance measure along vertical centerline; distance along flat plate, FT
y	=	distance normal to flat plate, FT

Subscripts

m	=	indicates mean values
o	=	refers to values at inside wall surface
w	=	refers to values at outside wall surface
VHS	=	volume heat source
NO VHS	=	no volume heat source

Moduli

Nu	=	Nusselt modulus
Pr	=	Prandtl modulus
Re	=	Reynolds modulus

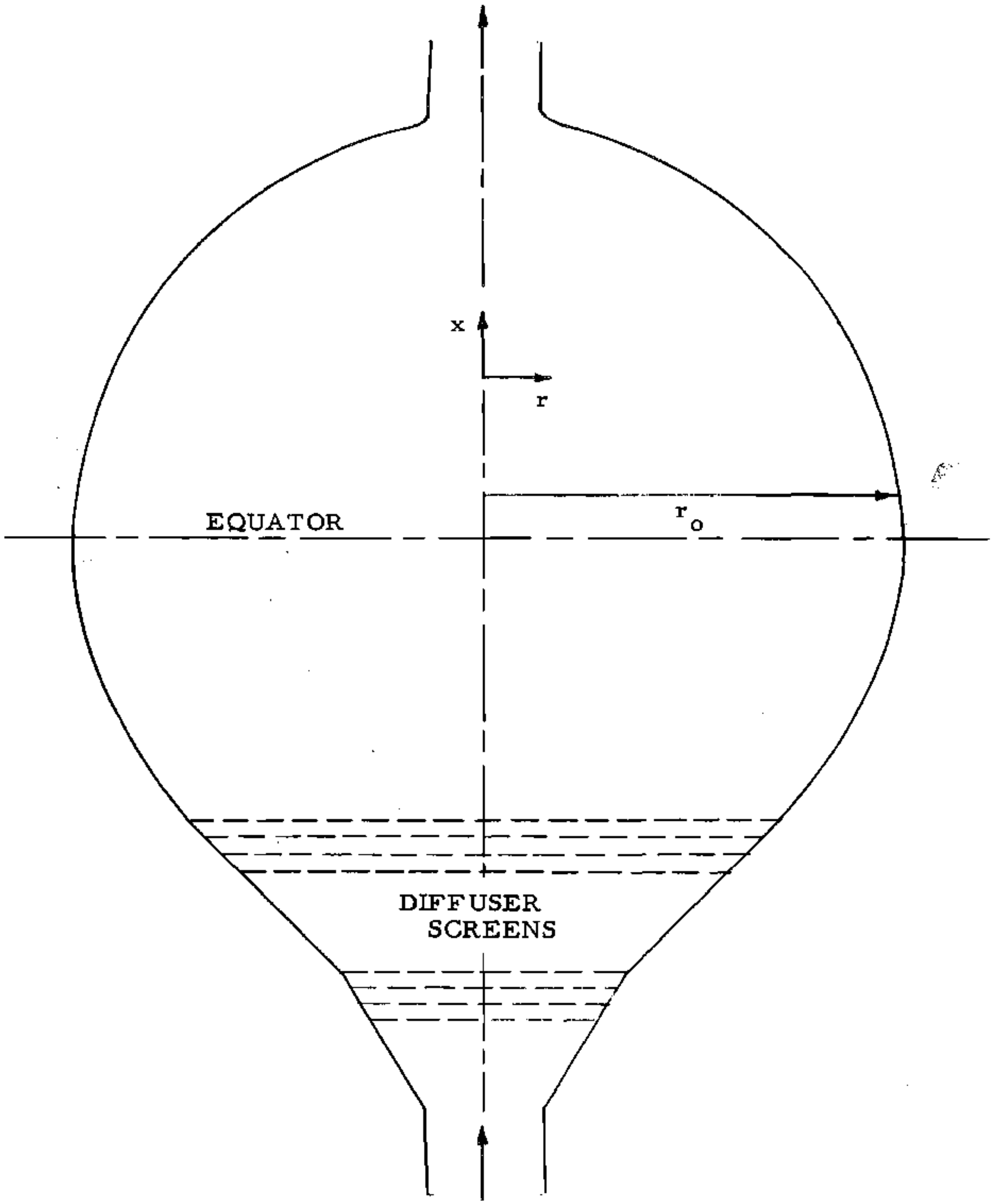

Figure 1. Sketch of the Core Vessel of the Homogeneous Reactor Test (HRT)

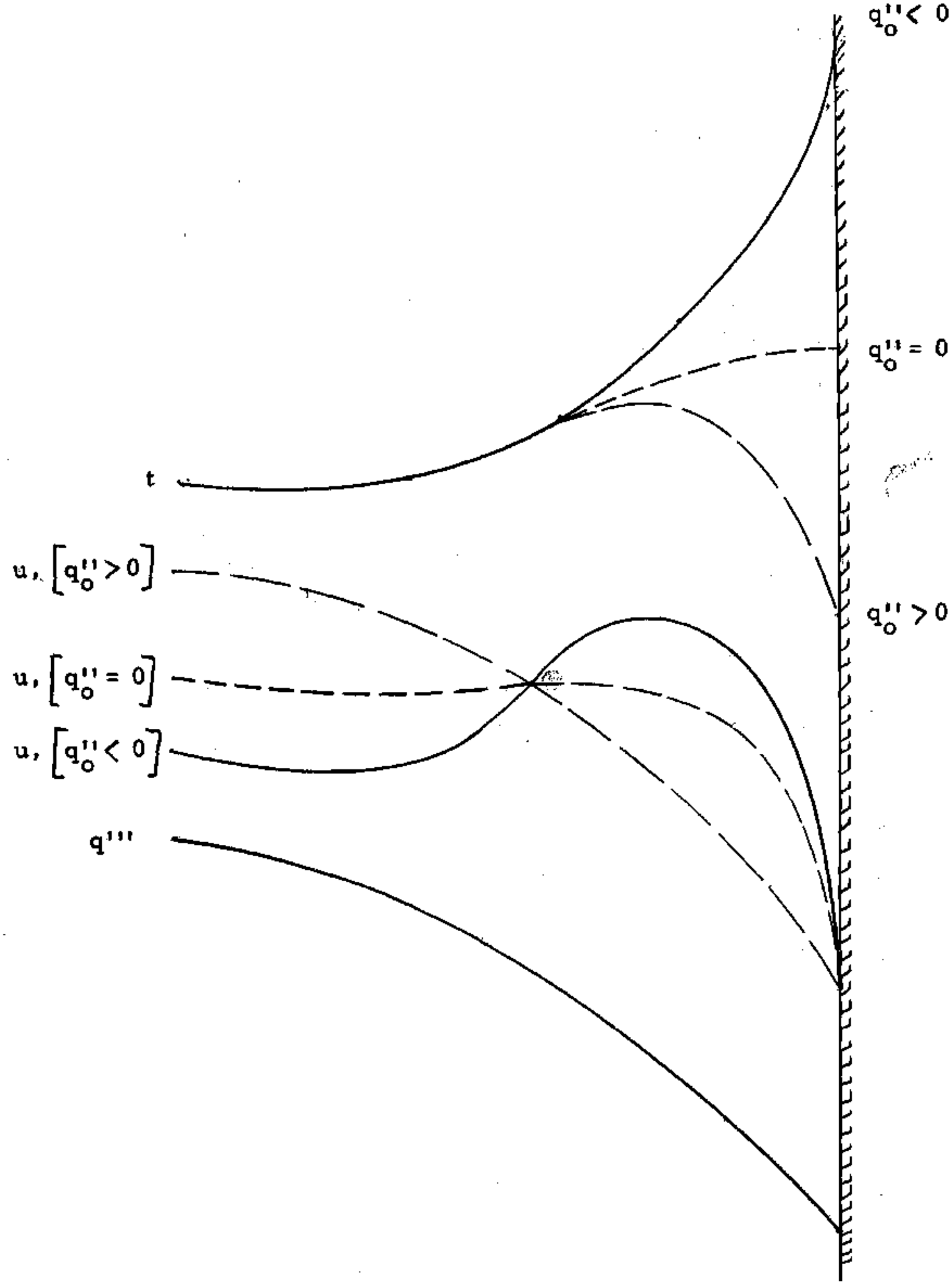

Figure 2. Possible Velocity, Temperature and Heat Source Profiles Near Core Wall (Upper Hemisphere)

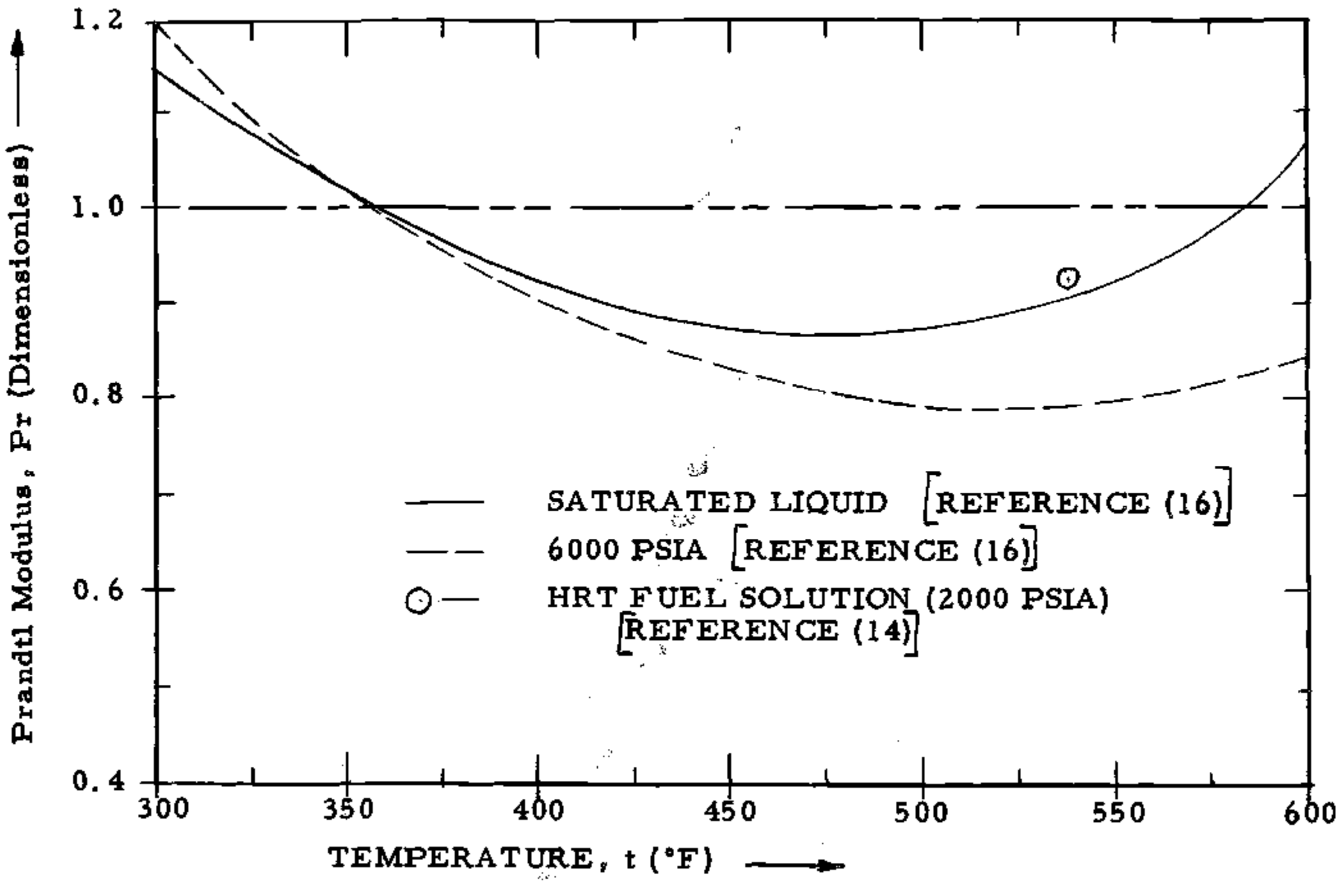

Figure 3. Prandtl Modulus Versus Temperature for Liquid H_2O

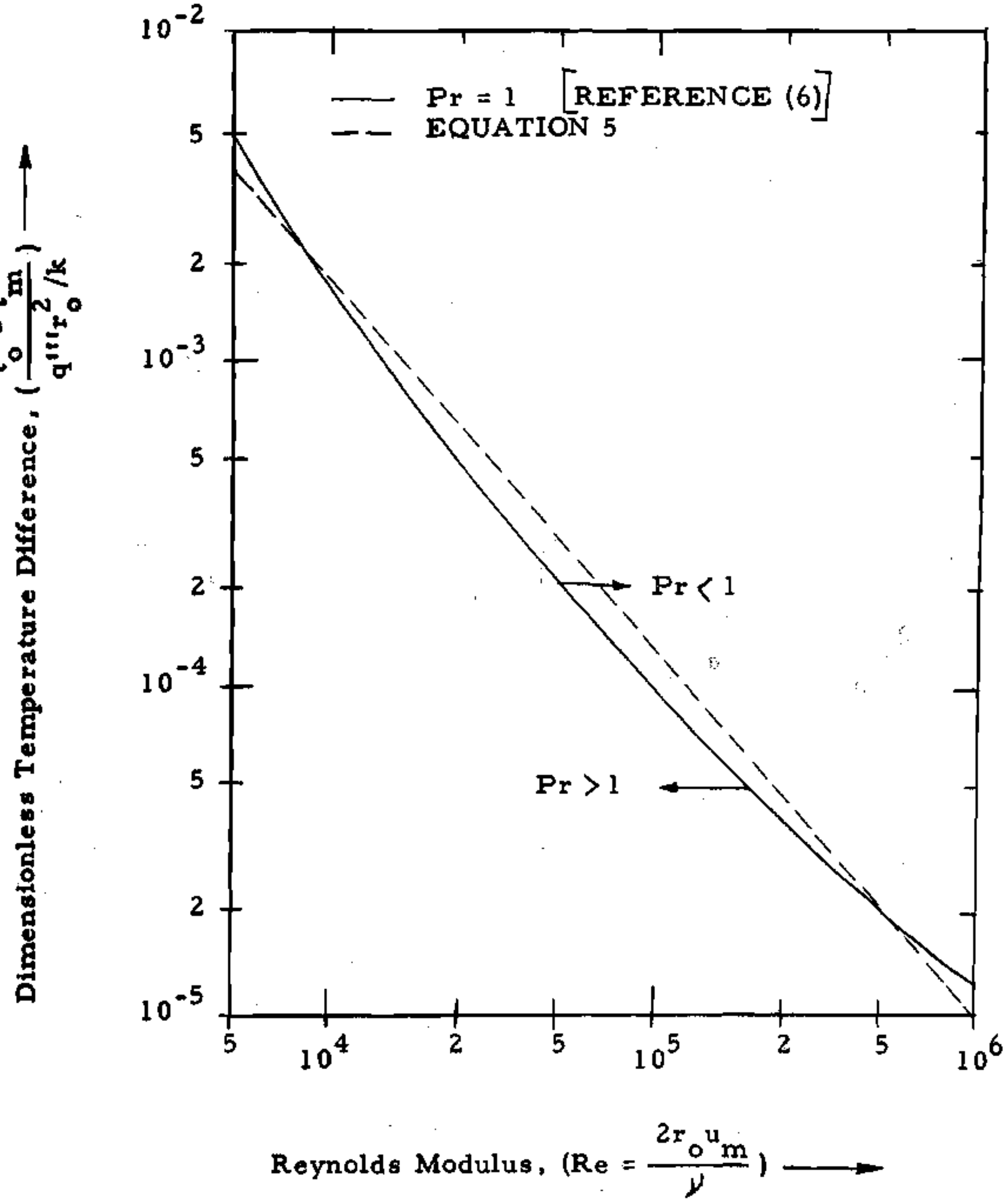

Figure 4. Dimensionless Temperature Difference Versus Reynolds Modulus

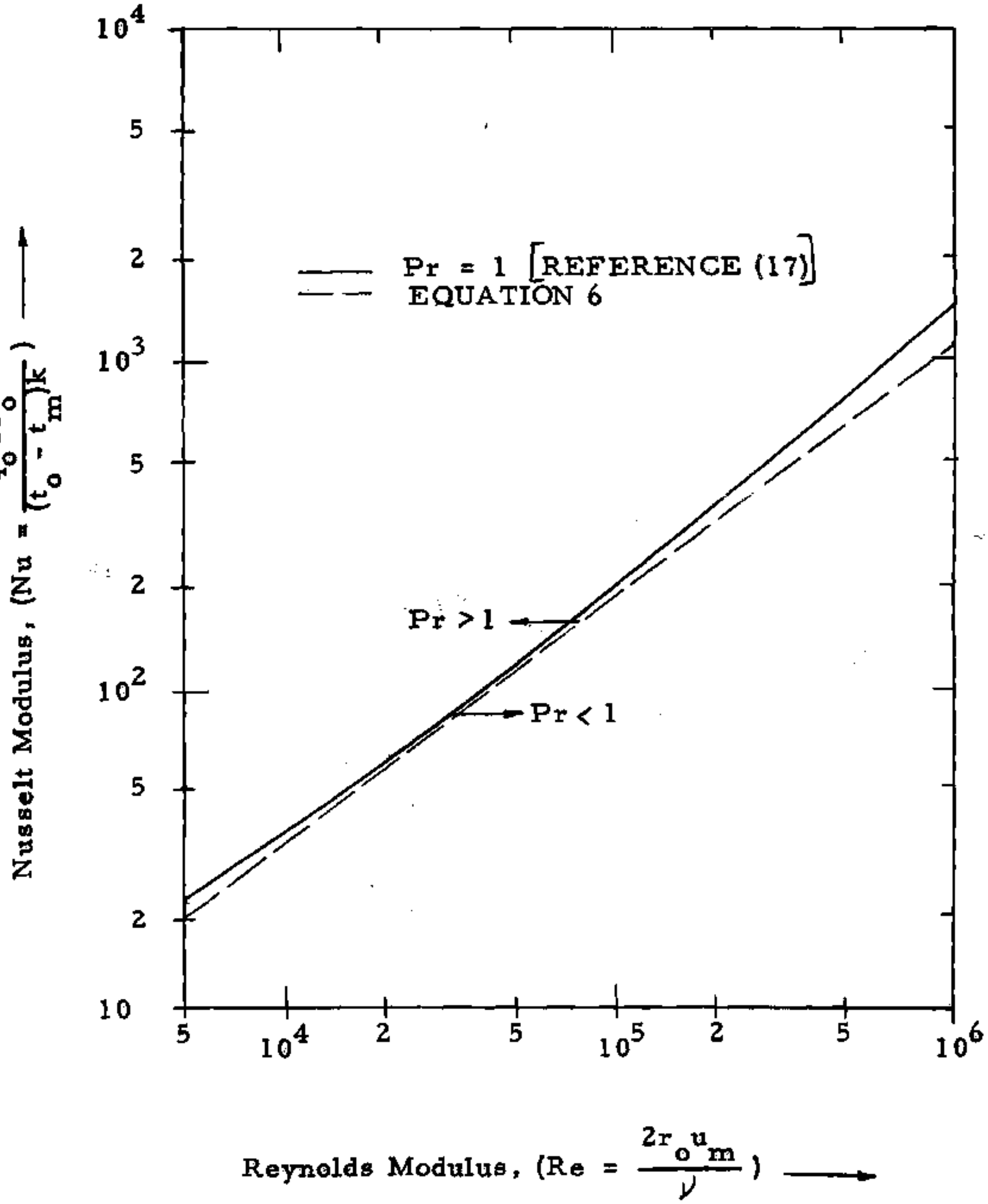

Figure 5. Nusselt Modulus Versus Reynolds Modulus

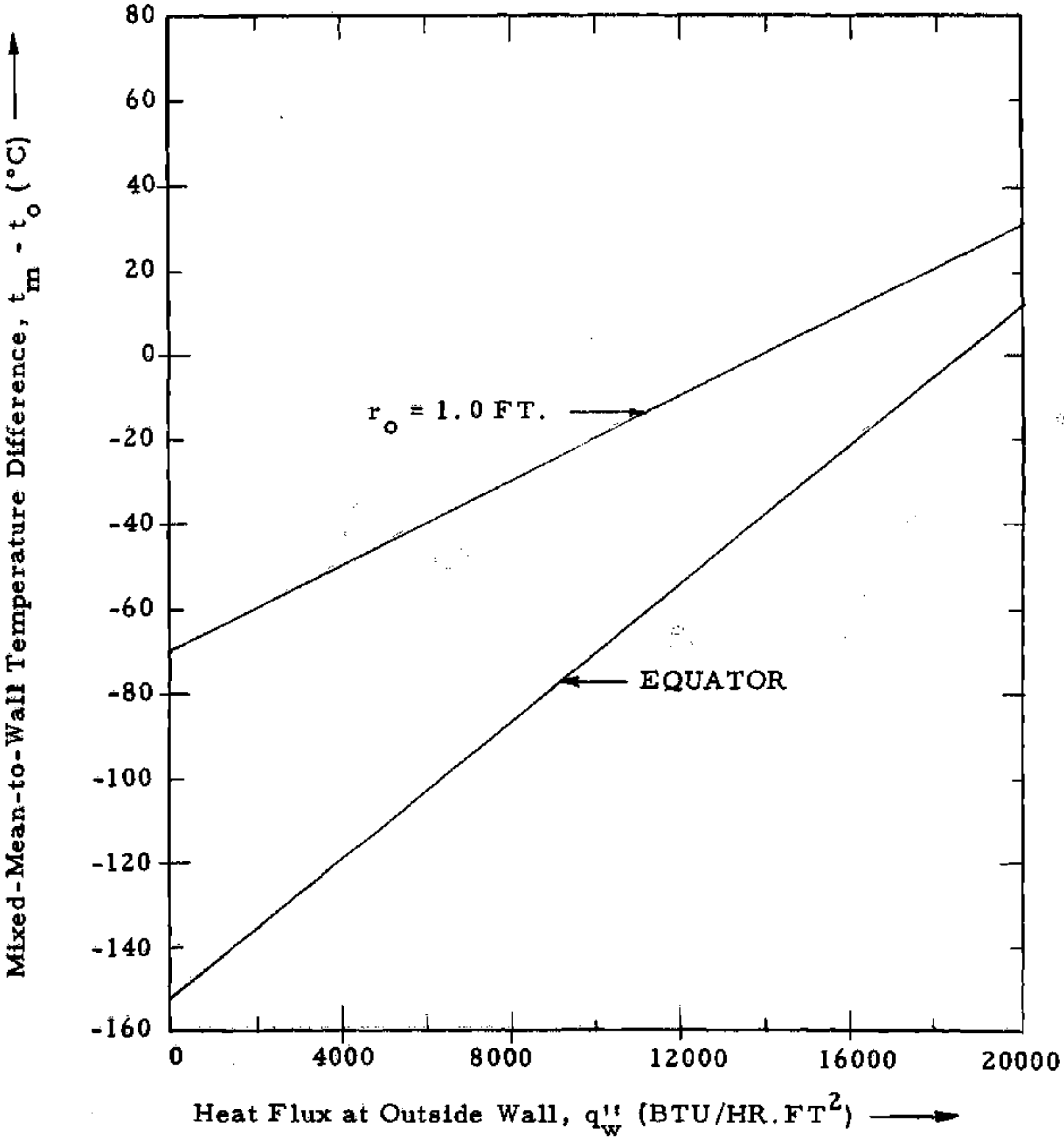

Figure 6. Temperature Difference Versus Heat Transfer Rate in Forced Convection

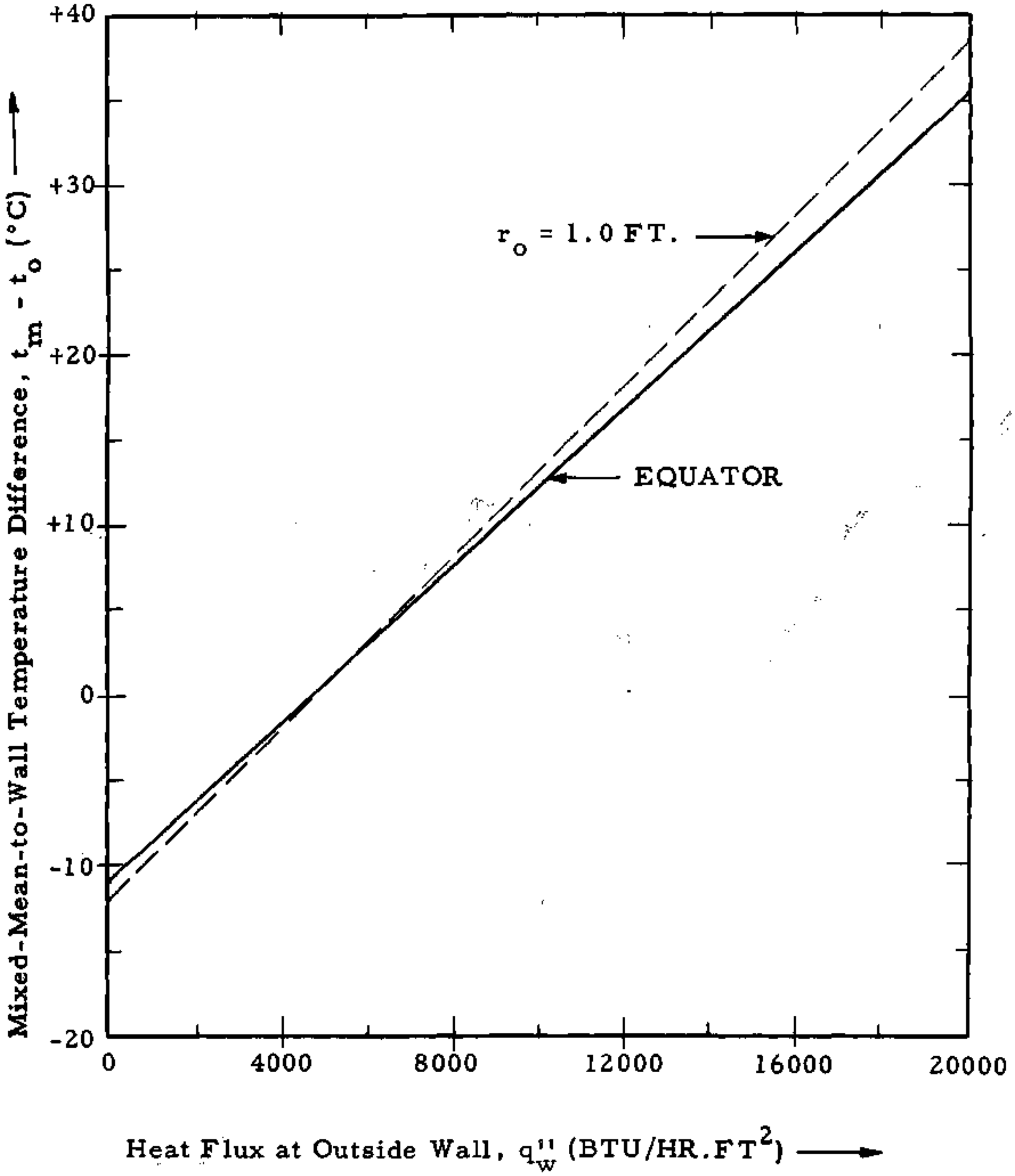

Figure 7. Temperature Difference Versus Heat Transfer Rate in Combined Free and Forced Convection

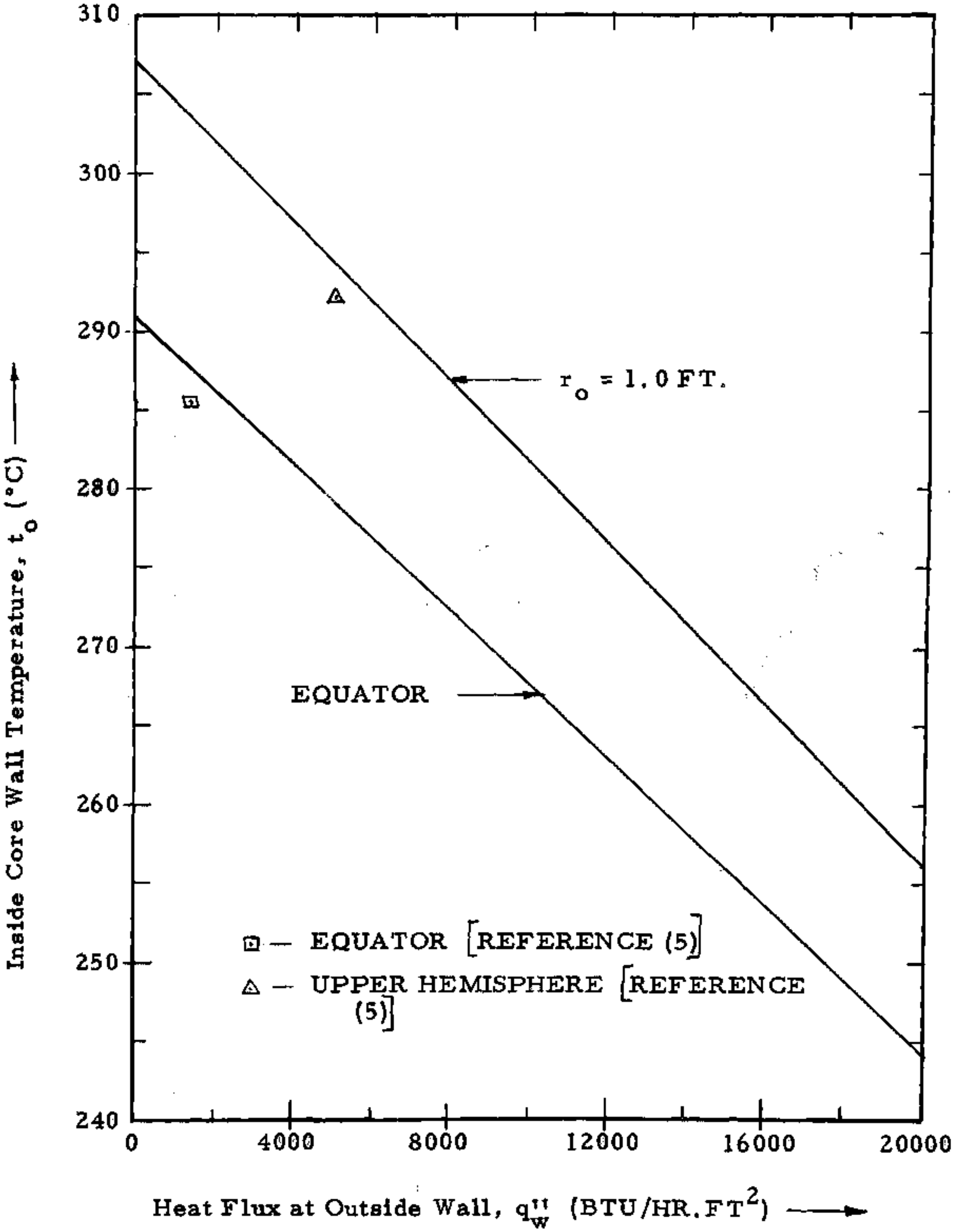

Figure 8. Inside Core Wall Temperature Versus Heat Transfer Rate at the Outside Core Wall

HRT Constants

Power (a)	5 MW
Heat Generation Rate in the Fluid at Core Wall (b)	$1.32(10^6)\frac{BTU}{HR.FT^3}$
Heat Flux at Wall due to Heat Generation in the Wall (c)	$3790\frac{BTU}{HR.FT^2}$
Core Flow Rate (494°F) (a)	400 GPM
Core Diameter (a)	32 IN.
Core Wall Thickness (a)	1/4 IN.
Core Wall Material (a)	Zircaloy -2
Thermal Conductivity of Zircaloy -2 (500°F - 700°F) (c)	$8.1\frac{BTU}{HR.FT-°F}$
Core Fluid Properties (Operating Conditions)	
Pressure (a)	2000 PSIA
Temperature (Avg.) (a)	536°F
Thermal Conductivity (a)	$0.35\frac{BTU}{HR.FT-°F}$
Viscosity (a)	$0.26\frac{LB}{HR.FT}$
Density (a)	$53.0\frac{LB}{FT^3}$
Specific Heat (a)	$1.24\frac{BTU}{LB.°F}$

(a) Reference (14)

(b) Reference (15)

(c) Reference (5)

<u>Sample Calculations</u>.--For forced convection at the equator of the core and an outside wall heat transfer rate of 8,000BTU/HR.FT^2

$$Re = \frac{2r_o \quad u_m}{\nu} = \frac{(2)(\frac{16}{12})(53.0)(0.17 \times 3600)}{0.26}$$

$$= 3.32 \times 10^5$$

$$t_m - t_o = 60 \frac{r_o}{k} Re^{-0.755} \left[q_w'' - Q'' - q''' r_o Re^{-0.375} \right] \qquad (9)$$

$$= 60 (\frac{16/12}{0.35}) (3.32 \times 10^5)^{-0.755}$$

$$\left[8000 - 3790 - (1.32 \times 10^6)(\frac{16}{12})(3.32 \times 10^5)^{-0.375} \right]$$

$$= -168°F$$

$$= \underline{-93°C}$$

For combined free and forced convection at the equator of the core and an outside wall heat transfer rate of 8,000 BTU/HR.FT^2,

$$F = 1 - \frac{2q_o''}{q''' r_o} \qquad (12c)$$

$$q_w'' = q_o'' + Q'' \qquad (8)$$

$$q_o'' = 8000 - 3790 = 4210 \text{ BTU/HR.FT}^2$$

$$F = 1 - \frac{(2)(4210)}{(1.32 \times 10^6)(16/12)}$$

$$= 0.9952$$

$$Gr = \frac{\beta r_o^4 q_o'' F}{\rho \nu^2 u_m c_p} \tag{12b}$$

$$= \frac{(1.52 \times 10^{-3} \times 4.17 \times 10^8)(16/12)^4(4210)(0.9952)}{(53.0)(4.9 \times 10^{-3})^2 (0.17 \times 3600)(1.24)}$$

$$= 2.68 \times 10^{12}$$

$$\lambda = (Gr)^{\frac{1}{4}} \tag{12a}$$

$$= (2.68 \times 10^{12})^{\frac{1}{4}}$$

$$= 1280$$

$$m = \frac{ber_o\lambda \ bei'_o\lambda - bei_o\lambda \ ber'_o\lambda}{ber_o^2\lambda + bei_o^2\lambda} \tag{12d}$$

$$= \frac{(0.9239)(0.3828) - (-0.3827)(0.9236)}{(0.9239)^2 + (-0.3827)^2}$$

$$= 0.7071$$

$$n = \frac{ber_o\lambda \ ber'_o\lambda + bei_o\lambda \ bei'_o\lambda}{ber_o^2\lambda + bei_o^2\lambda} \tag{12e}$$

$$= \frac{(0.9239)(0.9236) + (-0.3827)(0.3828)}{(0.9239)^2 + (-0.3827)^2}$$

$$= 0.7067$$

$$C = \frac{\lambda^2}{16}\left[\frac{\lambda}{n}\left(1 - \frac{1}{F}\right) + \frac{2m}{nF}\right] \qquad (12f)$$

$$= \frac{(1280)^2}{16}\left[\frac{1280}{0.7067}\left(1 - \frac{1}{0.9952}\right) + \frac{(2)(0.7071)}{(0.7067)(0.9952)}\right]$$

$$= -6.84 \times 10^5$$

$$t_m - t_o = \frac{q'''r_o^2}{4k}\left[\frac{64C}{\lambda^4}\left(mn + \frac{2m}{\lambda} - 1\right) + \left(\frac{1}{F\lambda^4} - \frac{64FC}{\lambda^8}\right)\right.$$

$$\left.(2\lambda^2 n^2 - 2\lambda^2 m^2 + 8\lambda n)\right] \qquad (12)$$

$$= \frac{(1.32 \times 10^6)(16/12)^2}{(4)(0.35)}\left[\frac{(64)(-6.84 \times 10^5)}{(1280)^4}\right.$$

$$\left((0.7071)(0.7067) + \frac{(2)(0.7071)}{1280} - 1\right)$$

$$+\left(\frac{1}{(0.9952)(1280)^4} - \frac{(64)(0.9952)(-6.84 \times 10^5)}{(1280)^4}\right)$$

$$\Big((2)(1280)^2(0.7067)^2 - (2)(1280)^2(0.7067)$$

$$\left. + (8)(1280)(0.7067)\Big)\right]$$

$$= 13.6°F$$

$$= \underline{7.5°C}$$

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